NOVEL COMPACT MICROSTRIP RESONATORS WITH MULTIPLE 2-D HILBERT FRACTAL CURVES

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ABSTRACT: *Novel compact microstrip resonators consisting of multiple two-dimensional Hilbert pre-fractal curves are presented. Proposed resonators with fractal curves of the third and the fourth order achieve length reduction of over 64% and 81%, respectively, when compared to the conventional capacitively coupled resonators tuned to the same frequency. Configuration with fractal curves of the third order, fabricated in standard PCB technology, exhibits improvement of unloaded Q-factor within range from 15% to 90%, depending on the resonant frequency.*

Key Words: *Microstrip Resonators; Fractals; Photonic Bandgap.*

1. INTRODUCTION

Compact microstrip resonators and filters are important for the next-generation wireless systems. Traditional capacitively coupled microstrip configurations incorporate half-wave resonators and therefore occupy large substrate areas, especially in the case of low operating frequencies. Therefore, a number of solutions emerged employing complex geometries that allow electrically long line to be folded in a small area, [1]-[3]. In order to reduce dissipative losses caused by the current density increase in miniaturized components, these configurations were designed using superconductive materials. However, new solutions are sought, employing low-cost manufacturing procedures and materials.

Recently, a number of applications of different fractal curves emerged, mostly in the antenna design, [4], and metamaterial surfaces, [5]. Fractal shapes were also used to form a multilayer PBG structure, [6]. The unique property of fractal curves is that, after an infinite number of iterations, their length becomes infinite although the entire curve fits into the finite area. This space-filling property can be exploited for the miniaturization of microstrip resonators. Due to the technology limitations, such as minimal line width and spacing achievable by the fabrication process, fractal curves are not physically realizable. Pre-fractals, fractal curves of the finite order, are used instead.

In this paper, novel microstrip resonator is proposed, consisting of N two-dimensional (2-D) Hilbert pre-fractal curves embedded in the resonator. The proposed structure, called 2-D Hilbert resonator, was designed using fractal curves of the third and the fourth order. Both configurations of 2-D Hilbert resonator were simulated for different N, using values typical for standard printed circuit board (PCB) technology, and the results were compared to the conventional capacitively coupled resonator. To validate the results, 2-D Hilbert resonator with N=3 fractal curves of the third order was fabricated and measured.

2. DESIGN

The generation of 2-D Hilbert fractal curves is shown in Figure 1, for the first four iterations. Hilbert pre-fractal curve has the same length and occupies the same substrate area as 2-D meander constructed from the lines having same width and spacing. However, due to the particular shape of the Hilbert curve, size of turns is reduced compared to the meander, thus resulting in weaker coupling between the turns and, therefore, in the improved performances.

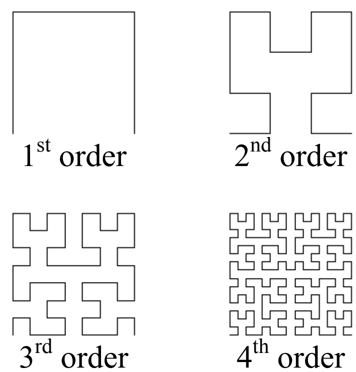
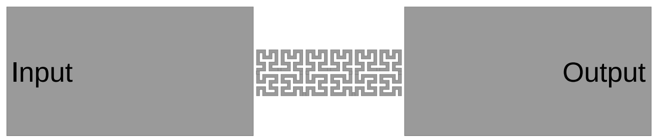


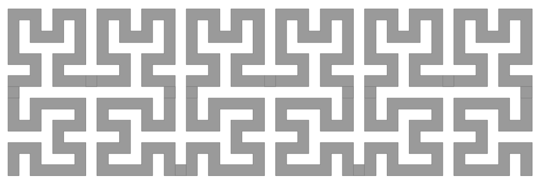
Figure 1 Generation of two-dimensional (2-D) Hilbert fractal curve, first four iterations

All configurations presented in this paper are based on the conventional capacitively coupled microstrip resonator with 50-Ohm feeding microstrip lines, and gaps equal to 125 um. Resonators were constructed on a 1.575 mm thick substrate having relative dielectric constant εr=2.17 and dielectric loss tangent equal to 0.0009.

In the proposed design, conventional microstrip resonator was replaced with *N* serially connected 2-D Hilbert curves of the third and the fourth order having line width and spacing equal to 125 um, as illustrated in Figure 2.



(a)



(b)

Figure 2 2-D Hilbert resonator with *N=3* serially connected fractal curves of the third order: (a) the whole structure, (b) enlarged detail

3. SIMULATION AND EXPERIMENTAL RESULTS

Performances of the proposed resonators were first determined using EMSight, EM simulator in Microwave Office. The proposed configurations were analyzed for different values of *N* and compared to conventional capacitively coupled microstrip resonator in terms of dimensions and unloaded and loaded Q-factor.

Simulation results for 2-D Hilbert resonators with fractal curves of the third and the fourth order, for various *N*, are shown in Tables 1 and 3, respectively, where *L* denotes resonator length, and

s_{21}^{0} is insertion loss at resonant frequency. Tables 2 and 4 show simulation results for conventional capacitively coupled microstrip resonators tuned to the same frequency as 2-D Hilbert resonators.

TABLE 1 Simulation results for 2-D Hilbert resonator comprised of *N* fractal curves of the third order

N	6	5	4	3	2
L, mm	**11.875**	**9.875**	**7.875**	**5.875**	**3.875**
f_r, GHz	3.01	3.5	4.2	5.27	7.06
s_{21}^{0}, dB	-6.18	-4.75	-3.73	-2.79	-2.43
Q_L	29	26	23	19	17
Q_U	**38**	**39**	**40**	**40**	**40**

TABLE 2 Simulation results for conventional capacitively coupled resonators tuned to the same resonant frequency as in Table 1

Resonator type	Conventional				
L, mm	**33**	**28**	**23.5**	**18**	**12.875**
f_r, GHz	3.03	3.52	4.12	5.27	6.97
s_{21}^{0}, dB	-5.33	-4.41	-3.73	-3.11	-2.74
Q_L	25	21	18	13	9.7
Q_U	**33**	**33**	**31**	**25**	**21**

TABLE 3 Simulation results for 2-D Hilbert resonator comprised of *N* fractal curves of the fourth order

N	3	2.5	2	1.5	1
L, mm	**11.875**	**9.875**	**7.875**	**5.875**	**3.875**
f_r, GHz	1.65	1.9	2.24	2.75	3.66
s_{21}^{0}, dB	-8.14	-6.82	-5.96	-4.9	-4.43
Q_L	26	25	24	23	25
Q_U	**31**	**31**	**32**	**34**	**39**

TABLE 4 Simulation results for conventional capacitively coupled resonators tuned to the same resonant frequency as in Table 3

Resonator type	Conventional				
L, mm	**64.25**	**55**	**45.5**	**36.75**	**26.5**
f_r, GHz	1.62	1.88	2.23	2.75	3.68
s_{21}^0, dB	-10.35	-9.12	-7.45	-6.02	-4.81
Q_L	51	43	36	31	20
Q_U	**56**	**49**	**44**	**41**	**30**

By comparing values in Tables 1 and 2, it can be seen that 2-D Hilbert resonators with fractal curve of the third order achieve length reduction of over 64% in all cases when compared to the conventional ones. At the same time, unloaded Q-factor is improved for 15% to 90%, depending on the resonant frequency. A comparison of simulated responses of 2D Hilbert resonator with N=3 fractal curves of the third order and a conventional capacitively coupled resonator tuned to the same frequency is shown in Figure 3.

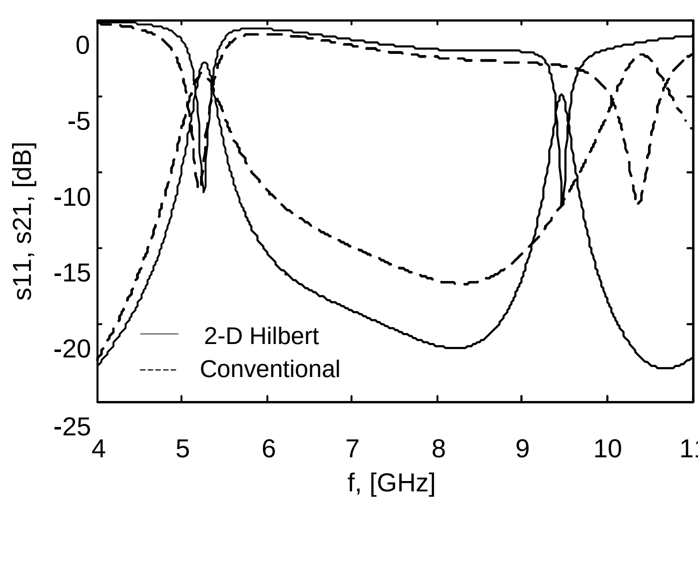
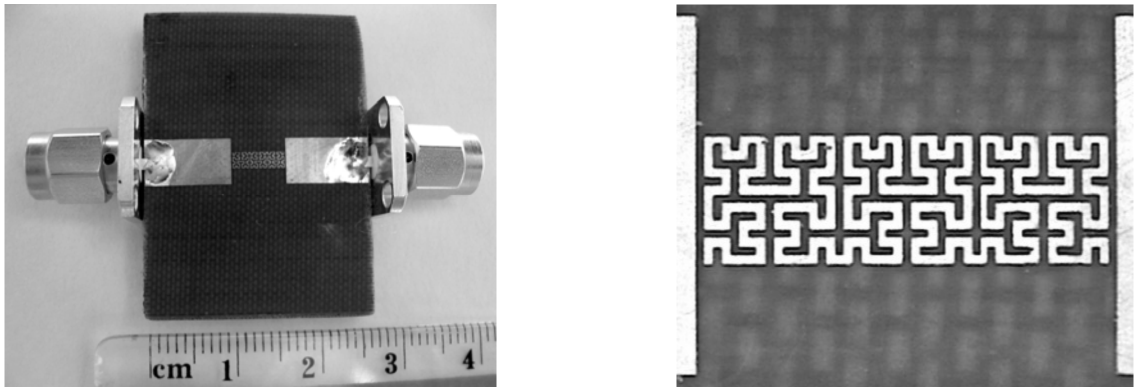


Figure 3 Comparison of simulated responses of 2-D Hilbert and a conventional capacitively coupled resonator

2-D Hilbert resonator with fractal curves of the fourth order, constructed using lines of the same width and spacing, is more then four times longer than its third-order counterpart, for the same overall length in the direction of the X-axis. Due to this, 2-D Hilbert resonators using fractal curves of the fourth order achieve length reduction of over 81% in all cases when compared to the conventional capacitively coupled resonators. However, due to the stronger coupling between the feeding lines and the resonator, 2-D Hilbert resonators with fractal curves of the fourth order achieve Q-factor improvement only for resonant frequencies above 3 GHz, as illustrated in Tables 3 and 4.

Multilayer configuration was also examined, where multiple 2-D Hilbert curves were placed in an additional conductive layer, 127 um bellow the microstrip resonator. Although showing the highest potential for length reduction (over 84%), multilayer configuration suffered from unacceptably high insertion losses, due to energy loss induced in the middle conductive layer.

To validate simulation results, 2-D Hilbert resonator with N=3 fractal curves of the third order was fabricated in standard PCB technology. Fabricated resonator with SMA connectors is shown in Figure 4a, while Figure 4b presents an enlarged detail showing good line definition achieved for 125 um lines and spacings.



(a) (b)

Figure 4 Photographs of the fabricated resonator with embedded N=3 2-D Hilbert fractal curves of the third order: (a) the whole structure, (b) enlarged detail showing good resolution of lines and spacings

By comparing simulation, Figure 3, and measurement results, Figure 5, a good agreement can be observed. Resonant frequency is shifted from 5.27 GHz (simulation) to 5.66 GHz (measurement). Since manufacturer specifications for substrate material allow εr variations in the range +/- 0.02 as well as variations of substrate thickness, this can be explained by the discrepancy between actual and simulated values of the dielectric constant and substrate thickness. Insertion loss equal to -3.32 dB was measured at the resonant frequency, in contrast to -2.79 obtained through simulations. This can be explained as the influence of SMA connectors used for measurement.

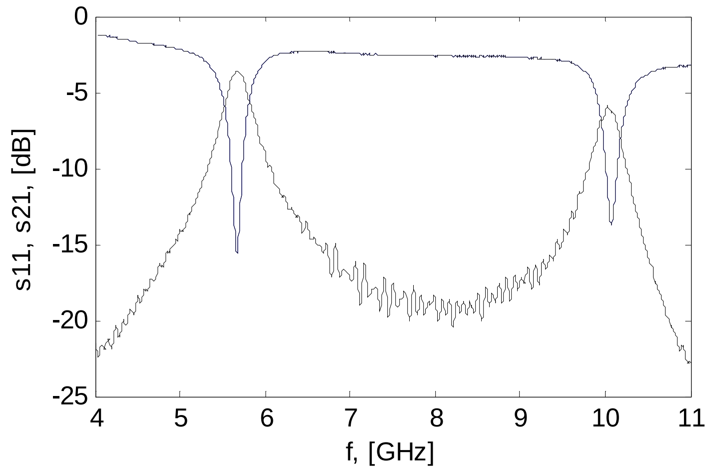


Figure 5 Measurement results for 2-D Hilbert resonator with *N*=3 fractal curves of the third order

Due to their planar nature, 2D Hilbert resonators could easily be redesigned for fabrication in superconducting technology.

4. CONCLUSION

In this paper, novel compact microstrip resonators with embedded multiple 2-D Hilbert pre-fractal curves were presented. Performances of proposed resonators were determined for different number of serially connected Hilbert curves (*N*) of the third and fourth order, and compared to those of the conventional capacitively coupled microstrip resonator. The proposed resonator using fractal curves of the third order achieved length reduction of over 64% for all values of *N*, while its unloaded Q-factor was improved for 15% to 90%, depending on the resonant frequency. Configuration with fractal curves of the fourth order achieved length reduction of over 81%, but suffered from strong coupling with feeding lines, thus showing improvement of the Q-factor only for resonant frequencies above 3 GHz.

Multilayer configuration of 2-D Hilbert resonator was also analyzed. Although exhibiting even greater potentials for length reduction (over 84%), it suffered from unacceptably high insertion losses.

Fabrication and measurements of 2-D Hilbert resonator using N=3 serially connected fractal curves of the third order validated superiority of the pre-fractal-shaped planar resonators over the conventional ones, both in Q-factor increase and in circuit size reduction.

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