Complementary Split Ring Resonators Using Square Sierpinski Fractal Curves

V. Crnojević-Bengin[1], V. Radonić[1], and B. Jokanović[2]

[1] Dept. of Electronics, Faculty of Technical Sceinces, University of Novi Sad, Serbia and Montenegro, phone + 381 63 552 162, E-mail: bengin@uns.ns.ac.yu, vasaradonic@eunet.yu

[2] Institute Imtel, Belgrade, Serbia and Montenegro, phone: + 381 63 330 999, E-mail: branka@insimtel.com

Abstract — **Split ring resonators and complementary split ring resonators are used in a left-handed media to obtain negative values of permeability and permittivity, respectively. In this paper, novel split ring geometry using square Sierpinski fractal curves is proposed, that reduces resonant frequency of the structure and achieves improved frequency selectivity in the upper transition band. In order to validate the results, left-handed microstrip line that uses square Sieprinski complementary split ring resonators is designed, fabricated and measured.**

Index Terms — **Left-handed metamaterials, Complementary Split Ring, Bandpass Filters, Fractals, Microstrip Resonators.**

I. Introduction

Recently, revolutionary results were obtained in the field of metamaterials, artificial structures that exhibit electromagnetic properties generally not found in nature. Among them, special attention has been paid to double-negative or left-handed (LH) media, that show simultaneously negative values of permittivity and permeabillity in a certain frequency range.

After proposing structure that exhibits negative permeability by decreasing the plasmon frequency into microwave range, [1], Pendry proposed split-ring resonator (SRR) as a unit cell of the structure that provides negative permittivity, [2]. Although having a narrow frequency range with negative permittivity, split-ring based configurations attracted a lot of attention, [3], [4]. Using a Babinet principle, complementary SRR (CSRR) was proposed, leading to a number of interesting filtering applications, [5], [6]. Since, CSRRs contribute to negative permittivity, adding gaps in the microstrip results with an LH microstrip line. In this structure, square (or circular) CSRRs are etched in the ground layer underneath the gaps, to achieve high magnetic coupling between line and rings at resonance. This structure behaves as a narrow band pass filter with a sharp transition in the lower band edge, However, it exibits poor frequency selectivity in the upper transition band.

Fractal curves are well known for their unique space-filling properties. Recent results showed that, due to the increas of the overall length of the microstrip line on a given substrate area as well as to the specific line geometry, using fractal curves reduces resonant frequency of microstrip resonators, and gives narrows resonant peaks, [7], [8].

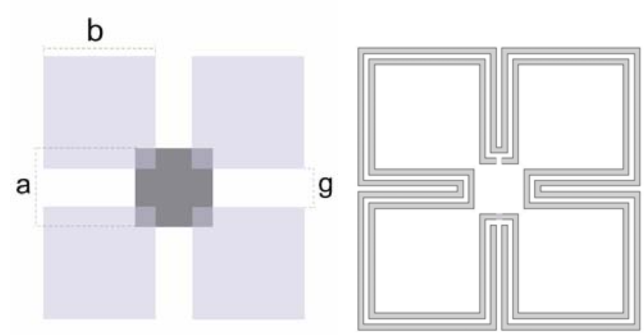
In this paper square Sierpinski fractal curves of the second order are used to substitute CSRRs in the LH microstrip line design. Proposed structure is simulated, fabricated and measured, and it's performances compared to those obtained by configurations using conventional square CSRRs of the same dimensions.

II. Configuration

Sieprinski fractal curve of the second order is shown in Fig. 1a, where *a*, *b* and *g* denoted size of it's segments. According to the rule of square Sieprinski fractal generation, following relations are obtained:

$b=3g$

$a=2g$



(a) (b)

Fig. 1. (a) Sierpinski fractal curve of the second order, (b) Sierpinski double split ring configuration.

Using the proposed fractal geometry, split-ring resonators were designed that follow the outter perimeter of the fractal curve. In order to increased the inductance of the split-rings, the line having minimal line width achievable by the conventional PCB technology was used, equal to 100 µm. Initial simulations showed that reducing *g* while keeping the overall dimensions of the fractal curve fixed, does not significantly influence it's resonance. On the other hand, this allows insertion of an inner split-ring, thus greatly increasing capacitance of the unit cell. Double square Sierpinski split ring configuration obtained in this manner is shown in Fig 1b.

In an LH microstrip line, split ring resonators are designed as complementary to those shown in Fig 1b, i.e. etched in the ground plane. Proposed LH line is depicted in Fig. 2 where both top (dark grey) and bottom (light grey) conductive layers are shown. The overall length of a single square Sierpinski CSRR is equal to 5.1mm, i.e. $\lambda/16$ on a given substrate.

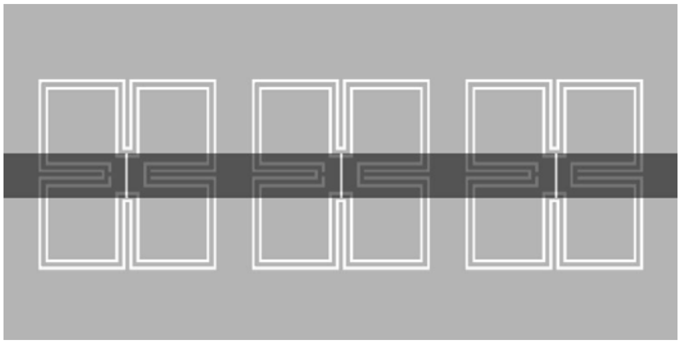


Fig. 2. Proposed LH microstirp line

Left-handedness of the proposed structure is evident from the comparison of phases of transmission coefficients obtained for a different number of unit cells *N*, Fig. 3. It can be seen that a phase advance exists in the passband, thus demonstrating backward propagation.

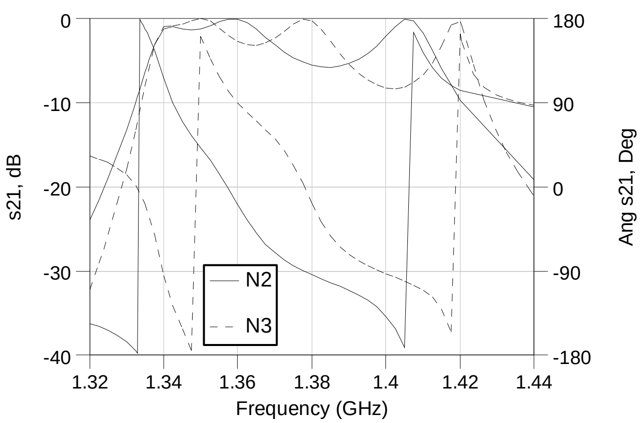


Fig. 3. Phase propagation for the proposed microstrip lines with *N*=2 and *N*=3 unit cells.

Proposed structure, as well as the one using conventional square CSRRs, can be characterized as the bandpass filter, whose order and dimensions can later be optimized.

In the case of conventional SRR or CSRR, adding multiple split-rings constructively influences the performances, due to the increased inductance and the capaticance of the structure. It is interesting to note that this does not hold for the case of fractal curves. Since fractal curves fill the space in an optimal manner, adding concentric split rings reduces the resonant frequency only for a few percents.

III. Simulation and Experimental Results

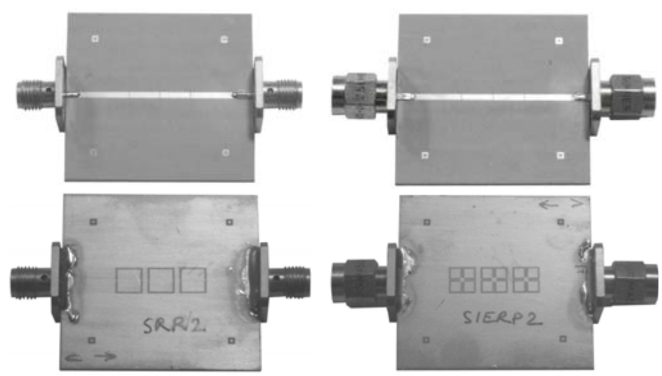
Performances of the filters were first determined using EMSight, EM simulator in Microwave Office. Responces of the proposed filter and the one using conventional square CSRRs having the same dimensions are compared in Table I for the lossy case, where *fc* denotes central frequency, *BW* is bandwidth, and s_{21}^{0} is insertion loss in the passband.

Table I. Comparison of simulation results for LH bandpass filters using square Sierpinski and conventional CSRRs

CRSS type:	**Conventional**	**Sierpinski**
fc [GHz]	2.123	1.365
BW [MHz]	170.95	69.5
BW [%]	8.05	5.9
s_{21}^{0}, [dB]	-5.62	-12.8

It can be seen that using Sierpinski-shaped CSRRs reduces central frequency for more than 35% for the same overall filter dimensions. This demonstrates potentials that fractal geometries of this type have for filter miniaturization.

To validate simulation results, LH microstrip lines using square Sieprinski and conventional square CSRRs were fabricated in standard PCB technology on a 1.27mm thick Taconic CER-10 substrate, having ε_r= 9.8 and dielectric loss tangent equal to 0.0025. Photographs of top and bottom layers of both fabricated structures are shown in Fig. 4.



(a) (b)

Fig. 4. Top (upper) and bottom (lower) sides of fabricated structures: (a) using conventional square CSRRs, (b) using square Sierpinski CSRRs.

Simualtion and measurement results for both structures are shown in Fig. 5. A good agreement with simulations can be observed, except for a shift in frequency approximately equal to 10% that occurred in both cases. Since manufacturer specifications for substrate material allow εr variations in the range +/- 0.5 as well as variations of substrate thickness, this can be explained by the discrepancy between actual and simulated values of the dielectric constant and substrate thickness. Measured insertion losses correspond well to the simulated ones.

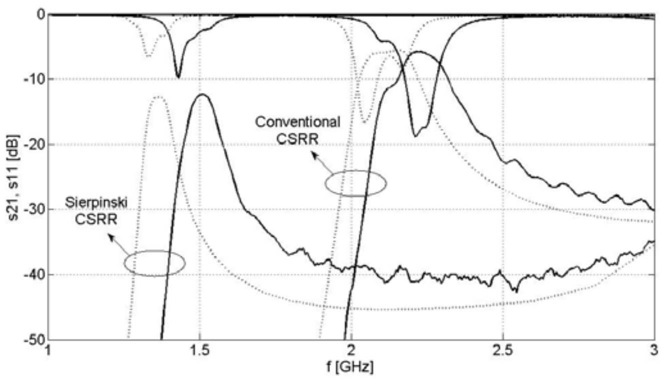


Fig. 5. Simulation (dotted line) and measurement (full line) results for lines using conventional square CSRRs and square Sierpinski CSRRs.

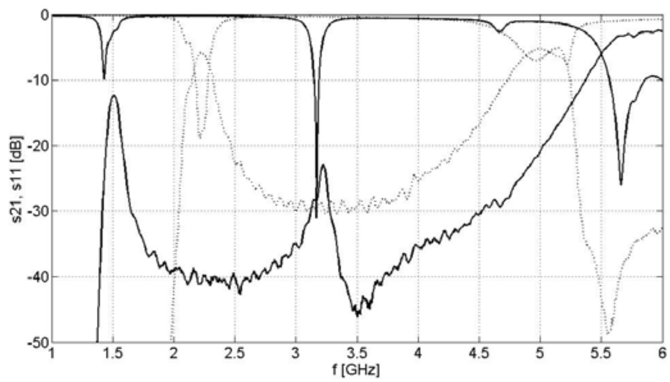


Fig. 6. Measurement results for lines using conventional square CSRRs (dotted line) and square Sierpinski CSRRs (full line) in a wide frequency range.

Fig. 6 shows measurement results for both structures performed up to 6GHz. It can be seen that the configuration using Sierpinski fractal curve shows sharp transition on both sides of the passband. Furthermore, it suppresses the second harmonic bellow 22dB, thus creating a wide and deep stopband.

V. Conclusion

In this paper complementary split ring resonators were presented that use square Sierpinski fractal geometry instead of the conventional sqare or circular one. Using the proposed CSRRs, LH microstrip line was designed and it's performances compared to those of a same line using conventional square CSRRs.

Fabrication and measurements showed that using fractal geometry reduced central frequency of the passband for more than 35% for fixed dimensions of all the elements of the unit cell, thus validating inherent potential of fractals for circuit size reduction. Configuration using Sierpinski fractal curve simultaneously achieved high frequency selectivity at both sides of the passband, thus indicating it's feasiblity for filtering applications. Furthermore, a wide and deep stopband is observed, due to the suppression oh the second harmonic for more than 22dB.

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